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On the fractal dimension of fracture surfaces of concrete elements

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The problem of the relation between the fractal dimension of a fractured surface and the fracture toughness expressed by the stress intensity factor is investigated. The theoretical conditions for such assumptions are discussed. Collected experimental results and new tests performed onconcrete specimens subjected to Mode II fracture seem to confirm that relation within the scope of materials tested and with certain necessary restrictions.

1. The fractal dimension of natural lines and surfaces

The development of the fractal characterization of irregular lines and surfaces was initiated by two books by Mandelbrot [1, 2], who introduced the fractal concept to many fields of the science.

Man-made objects have in most cases contours and surfaces that are linear or curvilinear, corresponding to Euclidean geometry in which points, lines, surfaces and volumes have topological dimensions represented by integers: 0, 1, 2 and 3, respectively. However, natural objects like coastlines, clouds and rock surfaces are irregular and composed of "mountains" and "valleys". It is easy to observe that at higher magnification further families of "mountains" and "valleys" appear. The effective length of such an irregular line and the effective area of an irregular surface seem to be related to the magnification and therefore cannot be determined in an objective way. At the discontinuities these curves and surfaces are non-differentiable.

The word "fractal" has been proposed by Mandelbrot [1] together with fractal dimension D which characterizes the irregularity of the fractal objects. The lines have non-integer values of D between 1 and 2. The irregularity of surfaces is expressed by their fractal dimension varying between 2 and 3. The more irregular is the object the higher is its fractal dimension. The fractal dimension exceeds the Euclidean one.

The fractal dimension is defined by the relation corresponding to a segment of a line from which the fractal line is generated:

$$D = \frac{\ln N}{\ln(1/r)} \quad \text{or} \quad N = (1/r)^D$$

where N is the number of sub-parts for which the initial segment of unit length is divided at each step and $1/r$ is the scaling factor, r being the length of each sub-part. The generation of two simple fractal lines is shown in Fig. 1.

The generation of a fractal line may be repeated indefinitely and the total length of the line is expressed as a function of r and D:

$$L = L_0 r^{-(D-1)} \qquad (1)$$

where L_0 is the length of the initial straight segment. The total length increases indefinitely with decreasing r, e.g. for $D = 1.5$ it is as given in Table I.

A more general relation for lines and surfaces has the form

$$L = L_0 E^{-(D_f - D)} \qquad (2)$$

where L and L_0 are the total and initial length and area, respectively, E is the scale of measurement, and D_f and D are the fractal and topological dimensions.

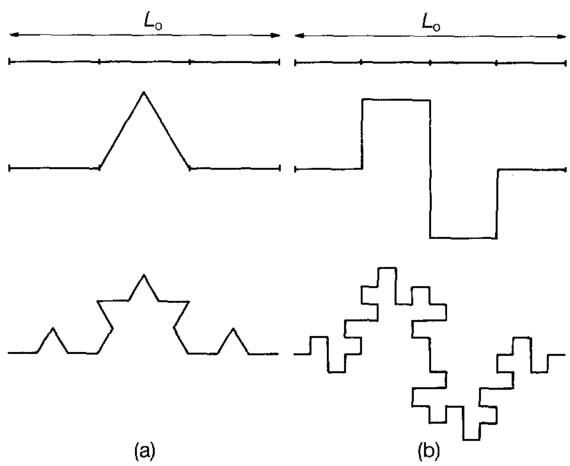

Figure 1 Generation of two examples of simple fractal lines. (a) $N = 4$, $r = 1/3$; $D = \ln 4/\ln 3 = 1.261\,86$. (b) $N = 8$, $r = 1/4$; $D = \ln 8/\ln 4 = 1.5$.

TABLE I Fractal line length (L) for $D = 1.5$

r	1	0.5	0.1	0.01	0.001	0.0001
L	1	1.41	3.16	10.00	31.62	100.00

Natural lines and surfaces with apparent irregularities are not necessarily fractal objects with a given fractal dimension. The necessary requirement is self-similarity, which means that within certain limits of magnification a larger region of the object should appear exactly or approximately similar to a smaller region regarded with appropriate magnification. In natural objects the self-similarity is not extended over all ranges of magnification: below and above certain levels the object may have not a fractal character. The reason is obvious: the nature of the objects recognized at different scales may be completely different.

The fractal dimension as a quantitative measure of the irregularity of the objects may be applied in various fields of materials science. Mandelbrot *et al.* [3] have shown self-similarity over two orders of magnitude for fracture surfaces of tempered steel and alumina. They also proposed a linear relation between the fractal dimension and fracture toughness for these materials.

2. The fractal dimension applied in the analysis of concrete elements

The irregular shape of cracks and fracture surfaces of cement-based composites was at the origin of the question whether they may have fractal dimensions like rocks and other natural objects. The application of fractal analysis is mostly aimed at a better understanding of the fracture mechanics of these materials by possible quantification of the fracture surface roughness.

There are many papers in which correlations between the mechanical properties of solids and fractal dimensions of fractured surfaces were investigated within different orders of magnitude of scale, and a few investigations were performed on concrete elements. Winslow [4] examined fracture surfaces of cement paste and has shown their fractal character, indicating its limits. Also a relation between water/cement ratio and the roughness of surfaces of fractured specimens was observed. Saouma *et al.* [5] studied concrete specimens with different maximum aggregate grain size from 20 to 75 mm, representing concretes used for the construction of ordinary structures and dams. They also observed that the fractal characters of concrete surfaces examined were without significant differences in fractal dimension for different aggregates. This last conclusion was attributed to the identical origin of all aggregates used in the specimens tested.

The roughness of a fracture surface is studied along a system of profiles. Their orientation is of great importance. In the investigations reported by Saouma *et al.* [5] the measurements were made with a specially designed profilometer which was displaced over the fracture surface along two families of orthogonal lines. Another technique is based on the execution of a polymer replica of the fracture surface. The replica is then sawn into slices by parallel sections. The contours obtained are called "fracture profiles" and are subjected to close examination and analysis: the length of each fracture profile is measured with a varying step r and the results are plotted in the logarithmic system of coordinates using Equation 1 in the form

$$\log L = \log L_0 + (1 - D) \log r$$

If the result obtained is approximately a straight line, it means that the fractal dimension (equal to its slope) is constant over certain levels of magnification. This is called the "vertical section" method. Another approach called the "slit-island" method is explained among others by Pande *et al.* [6] and was used for the analysis of fracture surfaces of metallic specimens.

To obtain a statistically reliable result of the analysis of a fracture surface by the vertical section method a large number of sections should be examined. Parallel sections executed, for example, along rectangular coordinates may give information about possible orthotropic properties of the fracture surface. In some cases random sections oriented at different angles may be studied.

The characterization of the fracture surface after the analysis of fracture profiles is based on an assumption that having determined the profile roughness parameter

$$R_L = L/L_0$$

it is possible to define the fracture surface roughness parameter

$$R_S = S/S_0$$

where L_0 and S_0 are the apparent projected length and area, respectively, and projection is on the mean or average topographic direction or plane (Fig. 2). Therefore, a relation of the following type is needed:

$$R_S = \overline{R_L \psi} \qquad (6)$$

where $\overline{R_L \psi}$ is an expected or average value of the product and ψ is the profile structure factor which expresses the position and orientation of each elementary segment of the fracture profile analysed.

Equation 6 is general and is not based on any assumption concerning the nature of the surface examined. The values of R_L and ψ are independent and should be obtained in a number of vertical sectioning planes differently oriented to assure the statistical representivity of the product; see Gokhale and

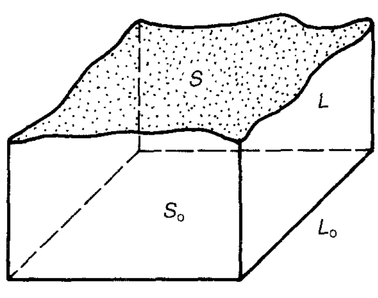

Figure 2 Symbols for irregular lines and surfaces with respect to their projections.

Drury [7], where an efficient sampling procedure is proposed for estimation of fracture surface roughness from the measurements. The profile roughness parameter R_L alone may not be correlated with the fracture toughness, and any correlations observed may be misleading [8].

However, an approximate formula was proposed by Underwood and Banerji [9] with the form

$$R_S = \frac{4}{\pi}(R_L - 1) + 1 \tag{7}$$

which provides, according to these authors, the best fit to experimental data. When comparing Equations 6 and 7 the following remarks should be made:

(i) R_S is an important quantity with respect to its possible relation to the fracture properties of the material, but it is inaccessible by simple experimental measurements;

(ii) R_L is easy to determine experimentally, but its relation to the surface roughness as given by Equation 7 is only approximate in view of Equation 6; and

(iii) Equation 7 has been proposed and discussed for metallic specimens and its application to cement-based materials has not yet been verified.

It should also be observed that R_L does not characterize completely and without ambiguity even a profile: different profiles may have identical values of R_L (e.g. Fig. 3). Also, two fracture surfaces with the same value of R_S are not necessarily similar.

Calculation of the fractal dimension D as well as of the fracture surface roughness parameter R_S is an interesting attempt to quantify the irregularities of natural fracture surfaces. Its application to the fracture mechanics of cement-based composites is promising. However, the quantitative conclusions from such calculations have only relative importance and should be limited to materials of the same kind. For example, the roughness of the fracture surfaces of a series of mortar specimens with variable w/c, age or grading curves may correctly reflect their fracture toughness. In contrast, any comparison between the fractal dimensions of cement pastes and concretes may give misleading results because of the different nature of these materials.

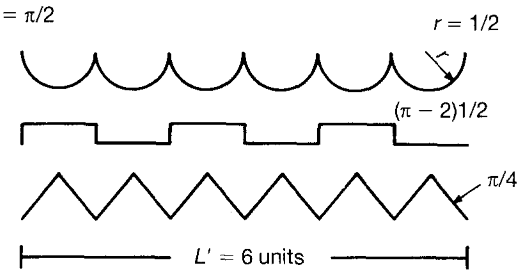

Figure 3 Three different profiles having identical values of the profile roughness parameter $R = \pi/2$ (after [10]).

The question of whether there is a general and reliable relationship between the fractal dimension of a surface and the fracture toughness of the material has been considered by several authors, but their answers are not completely concordant and require further investigations. The results for concrete elements fractured under combined compression and internal pressure [5] are compared in Table II with those obtained for other materials. The results show a relation between the fractal dimensions of the fractured surfaces and the critical values of the stress intensity factors K_{Ic} determined experimentally. The

TABLE II Mechanical properties and fractal dimensions for different materials

Authors	Material and test	D	K_{Ic} (MN $m^{-3/2}$)	γ(J/m^{-2})	Temperature (°C)
Saouma *et al.* [5]	Concrete (uniaxial compression)	1.07–1.12			
Underwood and Banerji [11]	Steel AISI 4030 (bending)	1.085	–	–	200
		1.091	–	–	300
		1.090	–	–	400
		1.072	–	–	500
		1.084	–	–	600
		1.079	–	–	700
Mecholsky *et al.* [12]	Alumina (bending)				
	UCC	1.15	2.5	–	–
	Lucalox	1.31	4.0	20	–
Anstis *et al.* [13]	Alumina (tension)				
	AD 90	1.21	2.9	11	–
	AD 999	1.31	3.9	19	–
Neilson [14]	Aluminosilicate GA TECH (bending)	1.18	2.2	–	–
Hellmann [15]	Alumina (tension)				
	WESGO (A 1500)	1.20	3.6	23	–
	GEND	1.23	3.9	27	–
Mecholsky [16]	Glass–ceramics (tension)				
	Zinc–silicate 1	1.05	1.6	22	–
	Zinc–silicate 2	1.09	1.8	22	–
	Zinc–silicate 3	1.11	2.2	27	–
	Lithia borosilicate	1.18	2.7	40.5	–
Mecholsky and Mackin [17]	Ocala chert (bending)	1.32	1.55	–	20
		1.26	1.46	–	300
		1.24	1.25	–	400
		1.15	1.05	–	500

observed relation may be expressed as follows: in a group of similar materials, those with higher values of K_{Ic} have a higher fractal dimension for their fracture surfaces. This may be considered as an indication that the fractal dimension of materials reflects their fracture toughness. The results in Table II are incomplete and do not allow for more detailed discussion. As mentioned above, it is incorrect to compare mechanical data and fractal dimension for different materials, e.g. no conclusion may be formulated from the fact that alumina and Ocala chert have similar values of D and different values of K_{Ic}. Duxbury [18] indicated that the correlation between the fracture toughness and fractal dimension may be either positive or negative, and some additional specifications as to the materials compared are necessary before the formulation of conclusions of a useful character. In concrete specimens with an important pore system a negative correlation might be expected, and positive for a system of dispersed hard grains. The second possibility is typical for ordinary structural concretes.

The tests of concrete specimens subjected to Mode II fracture were aimed at a further investigation of relations between fractal dimension and roughness of the fracture surface after Mode II crack propagation.

3. Description of tests and measurements

The specimens were cast from concretes made with three kinds of coarse aggregate: crushed basalt, river gravel and crushed limestone. Other specimens were prepared with cement mortar and paste. The specimens were subjected to shearing as shown in Fig. 4.

The values of K_{IIc} were calculated according to the formula proposed by Watkins [19] :

$$K_{IIc} = \frac{5.11P_{cr}}{2Bb}(\pi a)^{1/2} \quad (8)$$

where P_{cr} is the value of the critical load P which initiated the crack propagation, b is the ligament depth, a is the notch depth and B is the specimen width (see Fig. 4 and Brandt and Prokopski [20]).

The fractured surfaces were used to prepare replicas with acrylic resin. The replicas were sawn into slices as shown in Fig. 5. Then, the profile lines of each replica were subjected to computer image analysis in a Magiscan (Joyce Loebl) and the length of each profile line was measured with varying steps equal to 0.45, 0.30, 0.15, 0.075, 0.05 and 0.0375 mm. The image of a profile line was transferred to a monitor and covered with a system of orthogonal lines giving 512 × 512 pixels. Modification of the scale of the profile lines caused a modification of the steps. To measure the length of the profile lines an erosion function was applied of unit width equal to 1 pixel.

Mean values from four measurements of profile lines were used to calculate the fractal dimension from Equation 1. Specimens of dolomite and gravel concretes were selected for fractal analysis from specimens with different values of K_{IIc}.

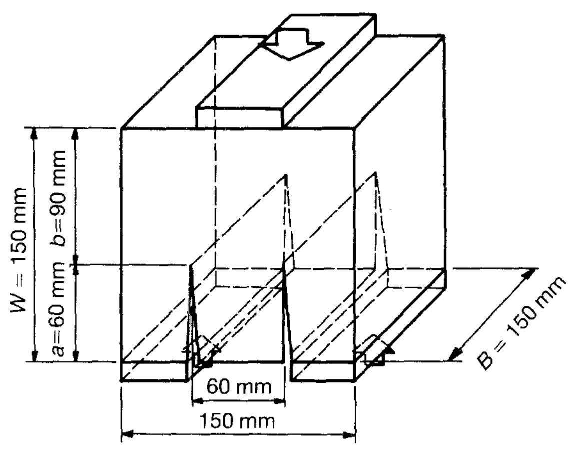

Figure 4 Specimens subjected to shearing in Mode II.

4. Test results and discussion

The results of tests are presented in Fig. 6 and Table III. The following conclusions may be drawn from the tests.

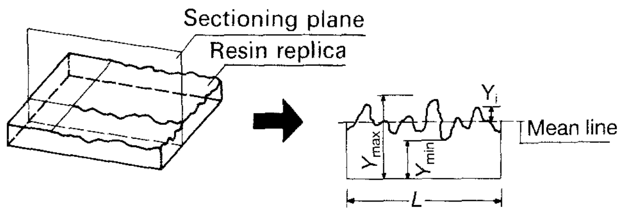

Figure 5 Preparation of replicas for analysis of profile lines.

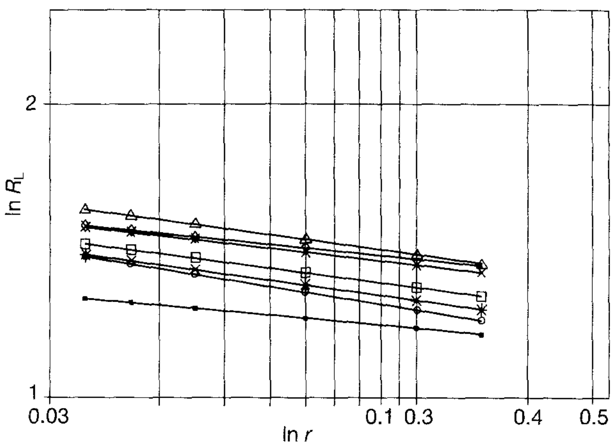

Figure 6 Fractal plots for profiles of specimens with different materials: (■) cement paste, (○) mortar, (★) dolomite concrete 1, (□) dolomite concrete 2, (×) basalt concrete, (◇) gravel concrete 1, (△) gravel concrete 2.

TABLE III Fractal dimension and fracture toughness

Material	D	K_{IIc} (MN $m^{-3/2}$)
Cement paste	1.033	1.60
Mortar	1.060	3.37
Dolomite concrete 1	1.050	3.90
Dolomite concrete 2	1.054	4.36
Gravel concrete 1	1.038	2.74
Gravel concrete 2	1.051	3.40
Basalt concrete	1.043	5.16

The fracture surfaces are fractal objects within the scope of the scales analysed and are characterized by fractal dimensions. The coefficients of exponential correlation for the lines in Fig. 6 are close to 0.99.

In this analysis it has been assumed that the profile lines analysis allows one to deduce the fracture surface roughness. It is therefore expected that the approximate Equation 7 may be used as characteristic for the comparison of surfaces of different materials, belonging however to the same group.

The relations between values of D and K_{Ic} confirm the previously mentioned conclusion. For Mode II fracture in cement-based composite materials it has been observed that higher fracture toughness is accompanied by higher values of fractal dimension of the fracture surfaces.

The differences in fractal dimensions for concrete, mortar and paste specimens may be explained by the influence of the smallest grains of sand and cement, which have a completely different nature from the composite materials.

The results obtained do not furnish any quantitative relations for practical applications and it is not yet possible to design cement-based composites with a given fractal dimension for a required fracture toughness. It seems, however, that new evidence has been obtained to support the hypothesis that the fracture surfaces of concrete-like composites are fractal objects. It is expected that this hypothesis may be of some practical importance in the future after further tests.

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